



## Early Journal Content on JSTOR, Free to Anyone in the World

This article is one of nearly 500,000 scholarly works digitized and made freely available to everyone in the world by JSTOR.

Known as the Early Journal Content, this set of works include research articles, news, letters, and other writings published in more than 200 of the oldest leading academic journals. The works date from the mid-seventeenth to the early twentieth centuries.

We encourage people to read and share the Early Journal Content openly and to tell others that this resource exists. People may post this content online or redistribute in any way for non-commercial purposes.

Read more about Early Journal Content at <http://about.jstor.org/participate-jstor/individuals/early-journal-content>.

JSTOR is a digital library of academic journals, books, and primary source objects. JSTOR helps people discover, use, and build upon a wide range of content through a powerful research and teaching platform, and preserves this content for future generations. JSTOR is part of ITHAKA, a not-for-profit organization that also includes Ithaka S+R and Portico. For more information about JSTOR, please contact [support@jstor.org](mailto:support@jstor.org).

Hence the author affirms that Dr. Robison's conclusions must henceforth receive a limitation.

Having shown that a positive advantage is obtained by the use of the circular conduit, amounting to about eleven per cent. of the total power, and that this value increases with an increase in the velocity of the wheel up to six feet per second, or more in large wheels, the author contends, that it is practicable to increase the efficiency of the best overshot wheels as now usually made, at least ten per cent. by this application. The only objections ever urged against the conduit were of a merely practical character, and the author shows that improved workmanship, and the modern use of cast iron, of which the conduit may be constructed, and provided with adjustments, render these no longer tenable.

Drawings of the apparatus used in these researches, and the tabulated results, were exhibited to the Academy.

---

Professor Lloyd read a paper "on the Phenomena of Thin Plates in Polarized Light."

The author stated, that his attention had been drawn to this subject by a letter which he had received from Sir David Brewster, describing a large class of hitherto unobserved phenomena. Sir David Brewster having expressed his desire, in this letter, to know whether the wave-theory could furnish an explanation of the facts which he had observed, Professor Lloyd was thus led to undertake the investigation which formed the subject of the present communication.\*

Mr. Airy had long since inferred, from a consideration of the form of Fresnel's expression for the intensity of reflected light, that when light, polarized perpendicularly to the plane

---

\* The present paper was read in the Mathematical Section of the British Association, last year; and a summary of the results was published in the *Athenæum*, of August, 1841. The author deferred submitting it to the Academy, in the hope of being able to add an experimental confirmation of some of the conclusions not noticed by Sir D. Brewster. He has, however, been compelled, by the pressure of other duties, to postpone still further this branch of the investigation.

of incidence, was incident upon a thin plate bounded by media of unequal refractive powers, a remarkable change in the reflected light should take place, when the angle of incidence was intermediate to the polarizing angles of the two surfaces of the plate. This theoretical anticipation was fully verified by experiment. When a lens of low refracting power was laid upon a plate of high refracting power, the rings which were formed appeared with a *black centre*, when the angle of incidence was less than the polarizing angle of the low refracting substance, or greater than the polarizing angle of the high refracting substance; while, when the incidence was intermediate to these two angles, the rings were *white-centred*, and the whole system was *complementary* to what it had been before. At the polarizing angle itself the rings disappeared, there being no light reflected from one of the surfaces of the plate, and therefore no interference.

The examination of this subject has since been resumed by Sir David Brewster; and he has repeated the experiments of Mr. Airy in a more general form, the incident light being *polarized in any plane*. He has thus been led to many new results. The rings are found to *disappear* under circumstances in which light is reflected from *both* surfaces of the plate; and there are many remarkable peculiarities in the transition of the rings into the complementary system.\*

It was to the theoretical explanation of these phenomena that Professor Lloyd now invited the attention of the Academy. In the conduct of the investigation he has generalized the methods followed by M. Poisson and Mr. Airy on the same subject. The incident vibration being resolved into two, one in the plane of incidence, and the other in the perpendicular plane, each portion will give rise to an infinite series of reflected vibrations, into which it is subdivided at the bounding surfaces of the plate. The expression of the resultant intensity, for each portion, being then deduced, the

---

\* The researches of Sir David Brewster are now published in the Philosophical Transactions for 1841.

actual intensity of the reflected beam is the sum of these intensities. Its value is found to be expressed by the formula

$$I = \cos^2 \gamma \frac{u^2 + 2uu' \cos a + u'^2}{1 + 2uu' \cos a + u^2 u'^2} + \sin^2 \gamma \frac{w^2 + 2ww' \cos a + w'^2}{1 + 2ww' \cos a + w^2 w'^2},$$

in which  $u$  and  $u'$  denote the ratios of the reflected to the incident vibration at the two surfaces of the plate, when the light is polarized in the plane of incidence;  $w$  and  $w'$  the corresponding quantities for light polarized in the perpendicular plane; and  $a$  the difference of phase of the successive portions of the reflected beam. The values of  $u$ ,  $u'$ ,  $w$ ,  $w'$ , are,

$$u = \frac{\sin(\theta - \theta')}{\sin(\theta + \theta')}, \quad u' = \frac{\sin(\theta' - \theta'')}{\sin(\theta' + \theta'')}, \quad w = \frac{\tan(\theta - \theta')}{\tan(\theta + \theta')}, \quad w' = \frac{\tan(\theta' - \theta'')}{\tan(\theta' + \theta'')}$$

where  $\theta$  denotes the angle of incidence on the first surface of the plate;  $\theta'$  the corresponding angle of refraction, or the angle of incidence on the second surface; and  $\theta''$  the angle of refraction at the second. The value of  $a$  is

$$a = \frac{4\pi}{\lambda} r \cos \theta';$$

$r$  being the thickness of the plate, and  $\lambda$  the length of the wave.

When the obliquity of the incident pencil is not very great, the squares and higher powers of  $u$ ,  $u'$ ,  $w$ ,  $w'$ , may be neglected in comparison with unity, and the expression of the intensity has the approximate value,

$$I = \cos^2 \gamma (u^2 + 2uu' \cos a + u'^2) + \sin^2 \gamma (w^2 + 2ww' \cos a + w'^2)$$

This quantity will be independent of the phase  $a$ , and therefore the intensity will be *constant*, and the rings *disappear*, when

$$uu' \cos^2 \gamma + ww' \sin^2 \gamma = 0;$$

that is, when the azimuth of the plane of polarization has the value given by the formula,

$$\tan^2 \gamma = - \frac{uu'}{ww'} = - \frac{\cos(\theta - \theta') \cos(\theta' - \theta'')}{\cos(\theta + \theta') \cos(\theta' + \theta'')}.$$

In this formula  $\cos(\theta - \theta')$  and  $\cos(\theta' - \theta'')$  are always positive;

and accordingly the resulting value of  $\tan \gamma$  will be real, and therefore the disappearance of the rings possible, only when  $\cos(\theta + \theta')$  and  $\cos(\theta' + \theta'')$  are of opposite signs; i. e. when the angles of incidence on the two surfaces are, in the one case greater, and in the other less, than the polarizing angle. The media at the two sides of the plate, therefore, must have different refractive powers.

Again, the *phases* of the two portions of the reflected beam, and which are polarized respectively in the plane of incidence and in the perpendicular plane, are given by the formulas,

$$\tan \alpha' = \frac{u'(1-u'^2) \sin \alpha}{u(1+u'^2) + u'(1+u^2) \cos \alpha},$$

$$\tan \alpha'' = \frac{w'(1-w'^2) \sin \alpha}{w(1+w'^2) + w'(1+w^2) \cos \alpha}.$$

The phases,  $\alpha'$  and  $\alpha''$ , are consequently in general different, and therefore the resulting light will be, in general, *elliptically polarized*. The author entered into some developments connected with this part of the subject, which does not appear to have been noticed by Sir D. Brewster in the course of his experimental inquiries; and he concluded by stating the important bearings which it may possibly have upon the phenomena of elliptical polarization by metals.

---

Professor Lloyd having, in the preceding communication, thrown out the idea that the elliptical polarization of metals might possibly be identified with that which is produced by a thin film on the surface of a reflecting body, Professor Mac Cullagh took occasion to observe that an analogous, but far more general, hypothesis had occurred to himself some years ago, among the various conjectures by which he had sought to account for the remarkable difference between the action of metals and that of transparent media in reflecting light. In his theory of crystalline reflexion he had found it allowable to suppose that the change in the elasticity of the ether, in passing out of

one medium into the other, takes place *abruptly* at their common surface; and he had thought it not unlikely that the supposition of a *gradual* change of elasticity, taking place within a *very small* space at one or both sides of the surface of a metal, might afford an explanation of the peculiar phenomena of metallic reflexion. Such a supposition would be mathematically equivalent to the hypothesis that an immense number of films, of which the refractive powers vary between given limits according to some law, compose a very thin stratum at the surface of a polished metal; and it would be in accordance with the inference drawn by Professor Mac Cullagh from certain formulas (Transactions R. I. A., vol. xviii. p. 70) that the law of equivalent vibrations is not observed in metals; an inference which, indeed, originally suggested to him the hypothesis in question. He had not yet compared the hypothesis with his formulas, but it was easy to see that it would explain the non-existence of an angle of complete polarization for metals, as well as the general fact of elliptical polarization; and perhaps the metallic brilliancy, difference of colour, &c. might be occasioned by the great number of reflexions in the variable stratum at the surface, and the endless variety of interferences produced by them.

The above was only one of the conjectures which had been formerly made by Professor Mac Cullagh in relation to this subject, and it was mentioned on this occasion chiefly on account of its analogy with the view taken by Professor Lloyd. Another and very different hypothesis, which was the first that had occurred to him, as being immediately suggested by the imaginary form which he had assumed for the velocity of propagation in a metal, will be found stated in the *Comptes Rendus* of the French Academy, tom. viii., p. 962, in a letter to M. Arago, dated May 11, 1839. It consisted in supposing the amplitude of the vibration within the metal to be proportional to a certain exponential of which the value is there given, accompanied with the remark that this expression for the vibra-

tion, if introduced into the differential equations (at that time unknown) which subsist at the confines of two media, would probably explain the peculiar phenomena of metallic reflexion, such as change of phase, &c. Very soon after that date the equations were discovered which hold good at the common surface of two *transparent* media (see Proceedings R. I. A. vol. i., p. 378); and it is certainly not a little singular that these equations, with the help of the aforesaid expression for the vibration, not only explain the change of phase, but lead to the precise formulas which had been previously given for the case of metallic reflexion (Transactions R. I. A. vol. xviii. p. 71). The application of the equations, however, to this case, cannot be regarded as legitimate without further proof; and the hypothesis is attended by another difficulty, the nature of which may be seen in the letter alluded to.

On the whole, Professor MacCullagh did not consider himself warranted, as yet, in choosing between his two hypotheses, nor even in concluding that one or other of them must be the right one. Before constructing any refined theory, he thought it necessary that the formulas to which he had referred, and which, if they are correct, must be the foundation of the theory, should be tested by experiments more accurate than any that had yet been made, and this was a task to which he hoped he should soon have leisure to devote himself.

Professor Lloyd explained, that the hypothesis which he had suggested had not been offered by him as an exact physical representation of the optical constitution of metals; but rather as one which lent itself, with tolerable facility, to mathematical expression, and the results of which might possibly, by a suitable determination of the constants of the formulas, be found to coincide with the phenomena, and therefore with the results of a more rigid theory.

---

RESOLVED,—That, in future, when the office of Secretary of the Academy is vacant, the vacancy shall be filled up by express election.